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CRITICAL TWO-PHASE FLOW

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Abstract—An overview of theories of critical flow, and the supporting evidence, is presented. It is argued that there is a diminishing return from increased theoretical complexity and that in many practical situations the problem is not sufficiently well defined to merit an elaborate approach. If basic scientific understanding is to be improved, detailed experiments will be required with control over certain conditions, such as the size and number of bubble nuclei, that are usually poorly known in practice.

INTRODUCTION

The purpose of this paper is to give a critical overview of the various analytical approaches that have been taken to two-phase critical flow. The topic has been studied for several decades and there exist numerous reports of theoretical and experimental studies. The recent review articles by Bouré (1977), Jones & Saha (1977), Lahey & Wallis (1975), Hutcherson (1975), Simon (1973) and Ardron & Furness (1976) discuss many aspects of the problem but without the breadth that is attempted here.

In order to present a general picture of this field and view the various parts of it in perspective it has been necessary to avoid detail, especially the repetition of long mathematical derivations. The reader who wishes to investigate specific questions should be able to find the answer in the cited literature. In particular, he will find specific expressions for critical flow rate resulting from the homogeneous equilibrium model, the frozen. Moody, Fauske, and Henry & Fauske models in the paper by Henry (1979), while a discussion of "two-fluid" or "separated flow" models appears in the paper by Wendroff (1979).

DESCRIPTION OF CRITICAL FLOW

The concept of critical flow is basically simple. A system (such as a nuclear reactor vessel) containing fluid under pressure is in communication with a receiver (such as the containment building) at a lower pressure via a flow path (such as the piping system and a broken pipe). Under critical flow conditions the flow rate from the system is independent of the conditions in the receiver. Usually one can identify a certain location in the flow path where the "critical" condition occurs and sometimes a mechanism, such as the bringing to rest of a perturbation, can provide the explanation for the lack of influence of downstream conditions (Bouré *et al.* 1975; Ardron & Duffey 1978; Prosperetti & van Wijngaarden 1976; Moody 1969).

The critical flow of a single-phase gas usually occurs when the Mach Number is equal to one at the smallest cross-section. Even though speeds are high, molecular relaxation phenomena are sufficiently rapid for the gas to be regarded as in thermodynamic equilibrium.

Two-phase critical flow is more complicated. Relaxation times for the formation of new interfaces (nucleation), heat, mass and momentum transfer, and the evolution of flow patterns are comparable with the time spent by the fluid in the "critical" region of rapid property change. Although it may be possible to define a mathematical condition of criticality at one location, an entire region (that may include parts of the upstream system) plays a role in determining how this condition is approached. Those readers who are familiar with the difficulties of making any two-phase gas–liquid flow situation "well-defined" will appreciate that we should perhaps not expect to be able to be too precise in our description of these phenomena.

MODELS FOR CRITICAL FLOW

Theoretical models for critical flow range all the way from the homogeneous equilibrium model (HEM), which is essentially a single-phase flow technique, to methods that attempt to

represent (at least approximately) all of the non-equilibrium phenomena. In between are many hybrid models that treat some of the non-equilibrium aspects by using assumptions or empiricism.

These models will be reviewed roughly in ascending order of complexity and evaluated for their practical usefulness.

HOMOGENEOUS EQUILIBRIUM MODEL (HEM)

This model of two-phase flow has been known for many years. The approach is to treat the two-phase mixture as a pseudo-fluid that can be described by the same equations as an equivalent single-phase flow (Wallis 1969a). The two phases are everywhere in equilibrium with equal velocities and temperatures; properties such as thermodynamic quality can be obtained from Steam Tables or some general "equations of state".

The HEM is the basis of several system codes, such as early versions of RELAP, for nuclear loss-of-coolant analysis. Critical flow can then be represented as a limiting case of the system code and is inherently compatible with it (otherwise there may be difficulties; for instance, a break flow computed from a non-homogeneous model may exceed the value for homogeneous critical flow somewhere upstream and numerical instabilities may arise as one tries to compute through this local critical flow condition).

The HEM is not a bad way of predicting the critical mass flux, G_c, in long pipes where there is sufficient time for equilibrium to be achieved and when the flow pattern is conducive to interphase forces that are adequate to repress relative motion (figures 1–3). Errors can be large

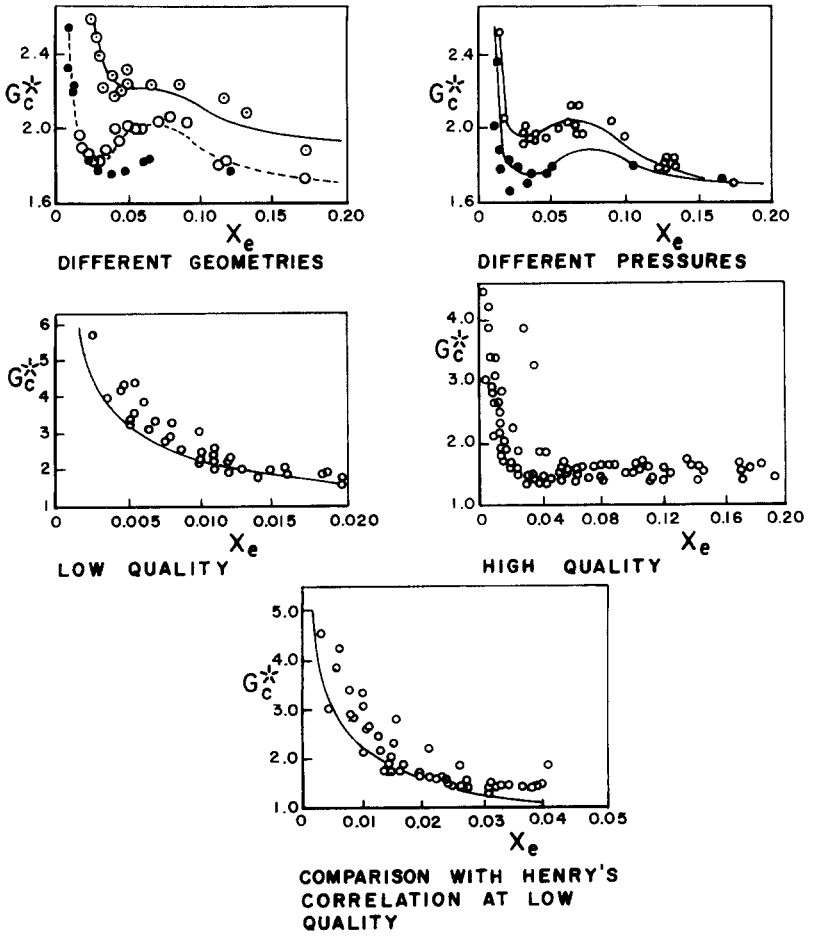


Figure 1. Ratio of critical flow rate to the predictions of homogeneous flow theory as a function of equilibrium quality. Some results of Henry (1968, 1970). (G_c^* = Mass flow/mass flow predicted by HEM; X_e = equilibrium quality).

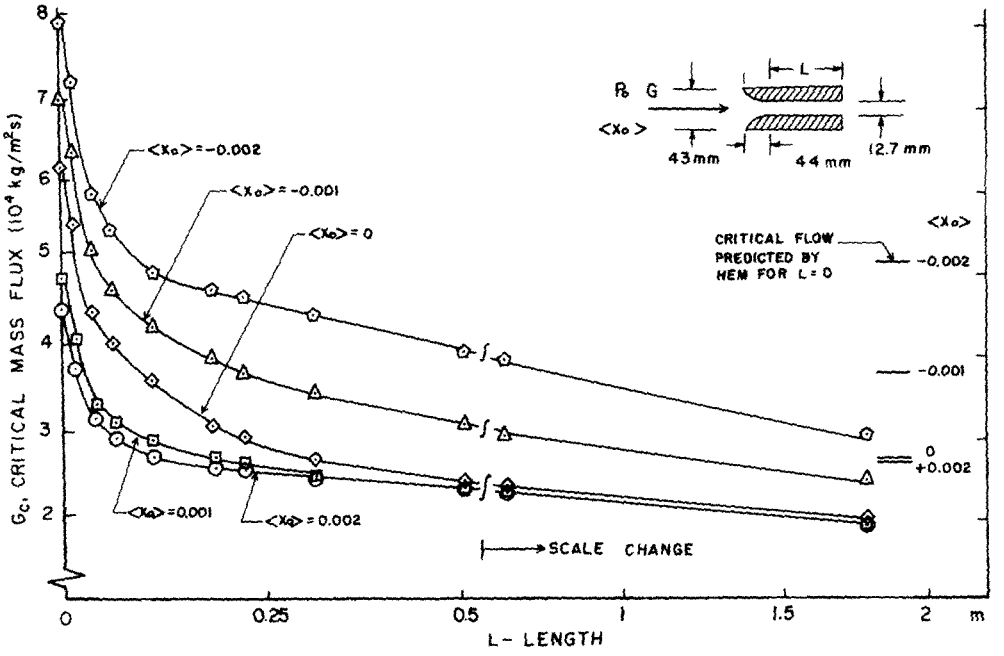


Figure 2. Effect of flow length on critical two-phase flow through 12.7 mm diameter tubes (Sozzi-Sutherland).

(a factor of 5 or so on flow rate) for short pipes, in which there is insufficient time for the vapor formation to proceed to equilibrium, and significant (but generally smaller: less than a factor of 2 on flow rate) in longer pipes if the flow regime, such as annular flow, allows large differences in phase velocities (Isbin *et al.* 1957; Henry 1968; Henry 1970a; Sozzi & Sutherland 1975).

A feature of the HEM is the discontinuity in fluid properties that occurs at the saturation line. Failure to account for this may explain some deviations between theory and experiment (Collins 1978). If sufficiently high single-phase liquid velocities can be achieved, critical flow occurs at the outlet of the pipe with the nucleation of the first bubble (Gutierrez & Lynn 1969; Wallis 1970). In the LOCA scenario it might be necessary to be careful about the transition from single-phase to two-phase choking during the early stages of a blowdown, particularly if the coolant temperature is abnormally low and nucleation is suppressed by non-equilibrium phenomena.

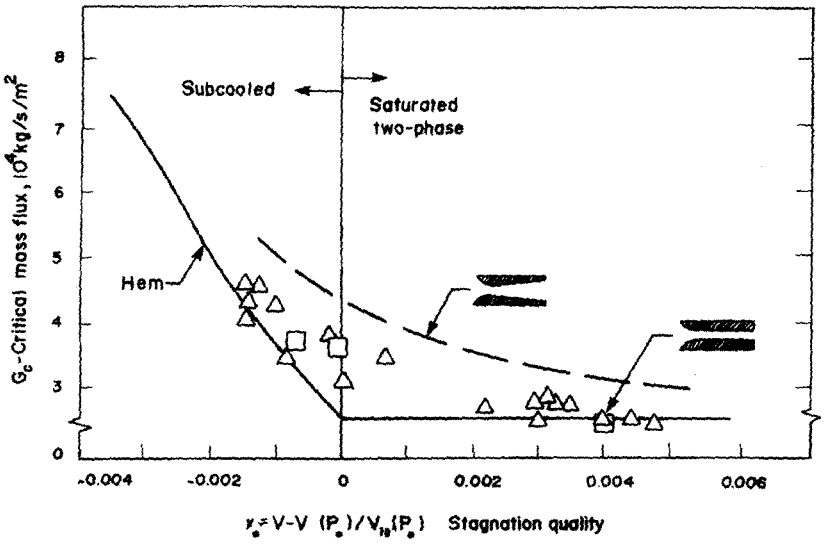


Figure 3. Critical flow in nozzles of various shapes (Morrison). ——— Venturi BHTD, 28 mm dia. throat, △ Long Throat, 13 mm dia. throat, $114 < L < 190$ mm, □ Long Throat, 28 mm dia. throat. $L = 127$ mm.

MODELS INCORPORATING LIMITING ASSUMPTIONS

Just as the HEM is based on the ideal case of complete interphase equilibrium, it is possible to derive a set of other models by making other limiting assumptions. This avoids the necessity for taking any account of the details of the non-equilibrium phenomena. The possibilities of this approach are limited and the following paragraphs describe most of them.

Frozen flow

For flow in short pipes or nozzles it is assumed that time is too short to allow any phase change. Thus the quality is kept constant throughout the expansion. Additional limiting assumptions, such as isentropic vapor expansion and equal phase velocities may also be made (Henry & Fauske 1971).

When the fluid upstream of the nozzle is subcooled liquid, this model implies that no vapor is formed at all and the emerging liquid velocity is given by Bernoulli's equation.

Slip flow models

Limiting assumptions can also be made about the relationship between the velocities of the phases. If the actual quality is assumed to be known (e.g. from an equilibrium or frozen flow model) one can treat the velocity ratio, ϵ, (mean vapor velocity/mean liquid velocity) as a variable and determine for what value of this parameter the overall mass flow will be a maximum. In situations where an energy balance determines the exit conditions (Moody 1965, 1966; Zivi 1964) the maximum flow rate occurs at a velocity ratio $\epsilon = V_G/V_L = (\rho_L/\rho_G)^{1/3}$, whereas if the momentum flux at the exit is known (Fauske 1962, 1963) the maximum flow rate occurs at $\epsilon = V_G/V_L = (\rho_L/\rho_G)^{1/2}$.

These limiting assumptions are the bases of the classical Moody and Fauske models that predict higher critical flow rates than would be obtained with the HEM.

It is also sometimes possible to allow velocity ratio to be a variable and to solve the momentum and energy equations simultaneously. Figure 4 shows the result of doing this for a particular case (Wallis 1969) assuming that flow into the nozzle occurs as saturated liquid and obeys Bernoulli's equation.

The two curves show the dependence of critical mass flux on velocity ratio, deduced by combining the continuity equation with either the energy or momentum equation for the control volume shown in the sketch. The maxima in these curves correspond to the predictions from the Fauske and Moody models. At the intersection point both momentum and energy are conserved. For the given conditions the predicted mass flux is rather insensitive to the assumptions (to predict critical flow we would also have to maximise G as the exit pressure was varied; the figure has been drawn for an experimentally determined exit plane pressure at choking; the quality, x, was derived assuming an isentropic equilibrium expansion). The HEM follows the energy equation since expansion can only occur in the converging nozzle and not in the straight pipe.

Isentropic streamtube model

Use of the velocity ratio as a parameter that allows maximisation of the flow rate offers no explanation of how (or if) this condition is actually achieved. In particular, it does not seem possible to relate the predicted velocity ratios to the dynamics of each phase and the interface boundary conditions in a way that is compatible with the other assumptions, such as absence of entropy generation. A "limiting assumption model" that is self-consistent is described by Wallis and Richter (1978) and by Wallis *et al.* (1976). It is based on an isentropic expansion of individual streamtubes that originate from the vapor–liquid interface as flashing progresses. There are no discontinuities in the velocity field and thermodynamic equilibrium is assumed. The overall flow rate is evaluated by integrating across the entire duct and is found to be a maximum, per unit area, at a certain downstream pressure, when inlet conditions are specified.

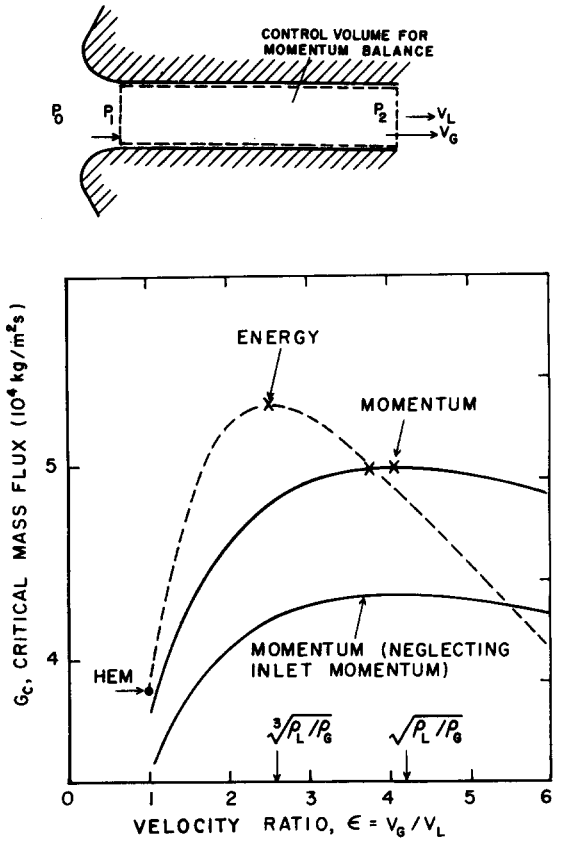


Figure 4. Predictions of various slip flow models, saturated water at 13.5 MPa expanding to 8.2 MPa, $x = 0.15$.

Summary

None of these limiting cases is necessarily inherently a "better" theory than any other and none of them provides a good representation of the details of the fluid behavior. However, they are useful as reference calculations that can be modified empirically to correlate data, much as isentropic cycle calculations in thermodynamics form a basis for rating real processes.

A comparison between the predictions of the two slip models, the streamtube model and the HEM model for water expanding from saturation pressure ($x = 0$) is shown in figure 5 and demonstrates the relative insensitiveness of the prediction to the assumptions, as long as thermodynamic equilibrium is achieved.

Figure 6 shows evidence that the predictions of several of these models are within the range of some experimental data. There are hundreds of figures in the literature showing comparisons of this sort and it is quite easy to pick out selected figures to favor the choice of a particular theory. Perhaps it would be worthwhile to undertake a systematic evaluation of all data to determine if quantitative criteria can be developed to guide the user who wishes to know how accurate a certain estimate is likely to be under various circumstances. This would be a major task in view of the many parameters that exert some influence on the behavior. An alternative strategy is to increase the level of complexity in the analytical model.

NON-EQUILIBRIUM MODELS

The major non-equilibrium phenomena that are ignored in the HEM are vapor bubble nucleation and interphase heat, mass and momentum transfer. Other processes that may influence behavior include multi-dimensional effects and developing two-phase flow patterns.

A wide spectrum of theoretical developments that incorporate consideration of some or all

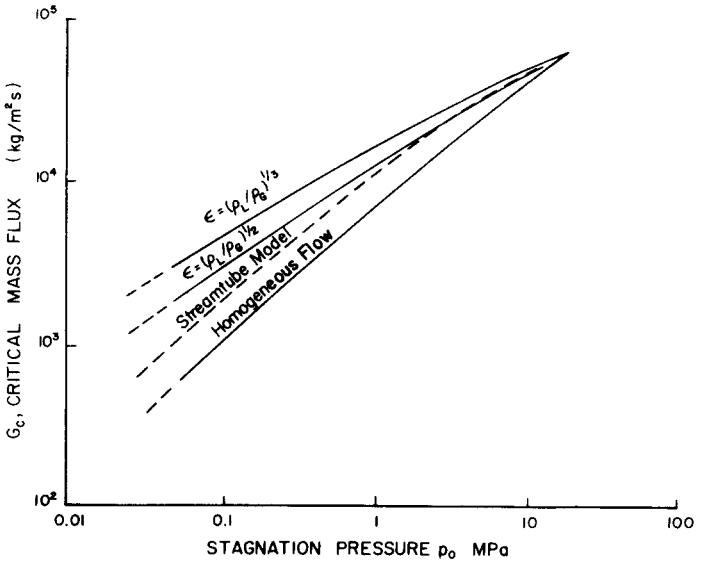


Figure 5. Critical mass flux vs stagnation pressure for saturated water at inlet, comparison between several "limiting case" models.

of these effects has appeared in the literature. Each approach contains empiricism in some form and it appears unlikely that any solution entirely from first principles will ever be possible or appropriate.

The various models range all the way from attempts to represent non-equilibrium with a single empirical correction factor (e.g. 60 or 84 per cent of the Moody prediction†) or function (e.g. Henry's correction factor in terms of equilibrium thermodynamic quality: Henry *et al.* 1970; Henry 1970b) to a complete set of one-dimensional equations of conversation of mass, momentum and energy for each phase coupled with criteria for nucleation and phase change.

To provide some order in the presentation of these models I have divided them into three categories:

(a) Empirical.

(b) Physically-based models for thermal non-equilibrium.

(c) Two-fluid (separated flow) models incorporating interphase interaction terms.

Empirical models

The Henry *et al.* (1970), Henry (1970b) and Henry & Fauske (1971) models typify this approach. Non-equilibrium vapor generation is handled entirely empirically by introducing a coefficient, N, that allows only a fraction of the equilibrium vapor generation to occur. The computed non-equilibrium vapor rate is then combined with a further simple assumption about the fluid flow (e.g. $V_L = V_G$) to predict the critical flow rate. This approach boils down in essence to a correlation of the deviation between the measured flow rate and the prediction from the HEM. By empirically relating the correction factor to the exit quality that would be achieved at equilibrium a considerable improvement in correlation of data from short tubes at low qualities (i.e. early in a blowdown) can be achieved (figure 7). The details of the flow path do not have to be worked out and one only needs to know the upstream conditions.

The Simpson & Silver (1972) model amounts to correlation of data by introduction of two empirical coefficients, A and B, to account for nucleation. Though a physically-based analysis is used to justify the correlation framework, the fitting of data is purely empirical. Equal phase velocities are assumed.

†Code of Federal Regulations, Title 10, Part 50, Appendix K, Acceptance Criteria for Emergency Core Cooling Systems for Light-Water-Cooled Nuclear Power Reactors (1976).

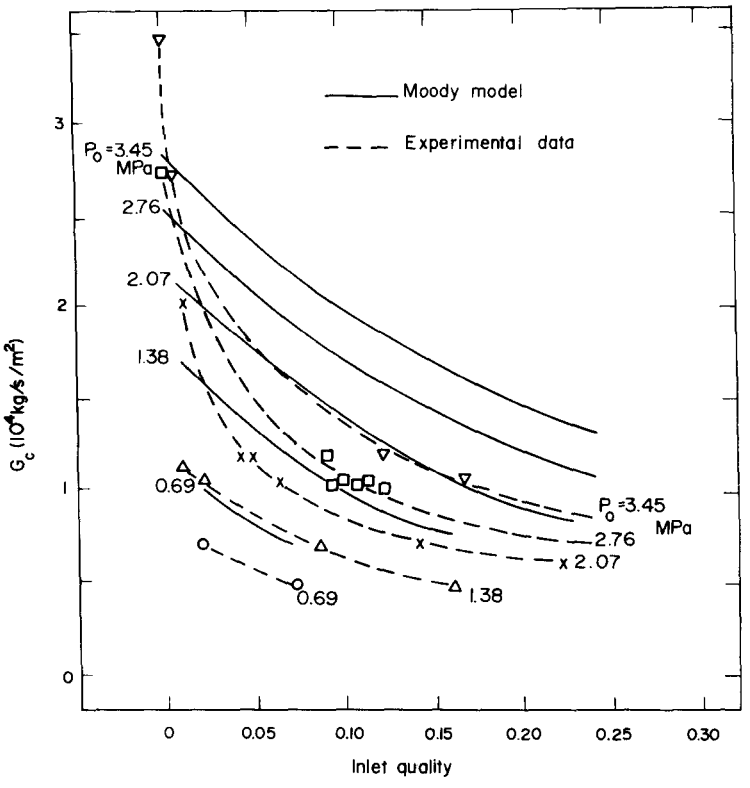
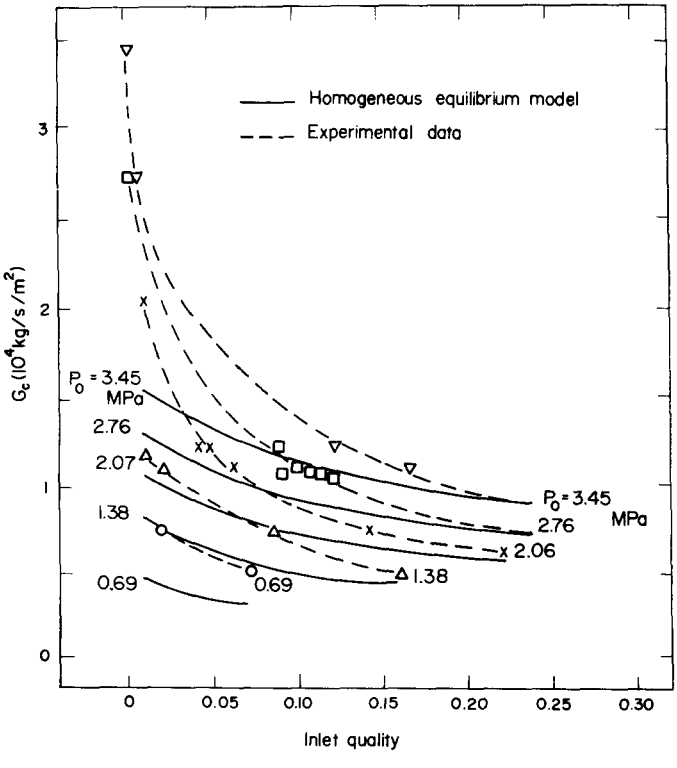


Figure 6(a).



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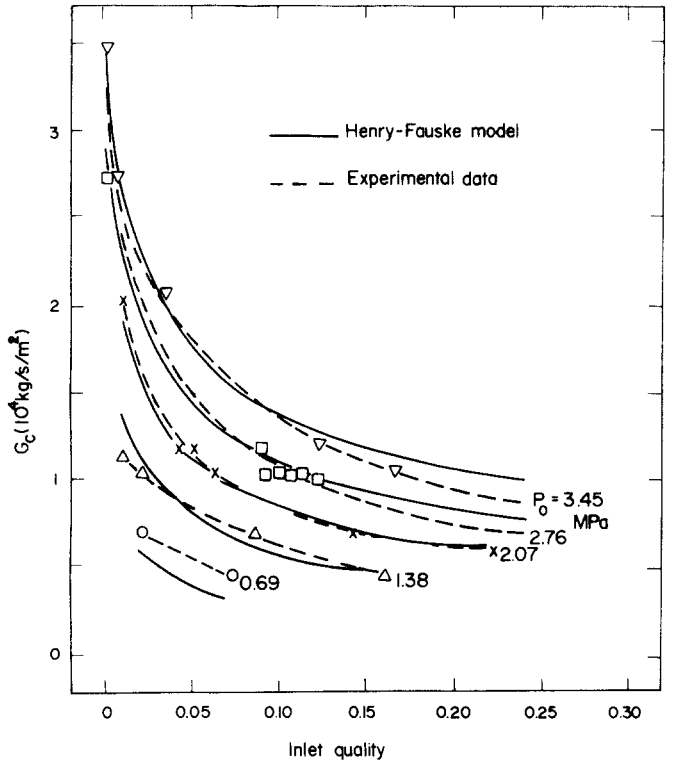

Figure 6(c).

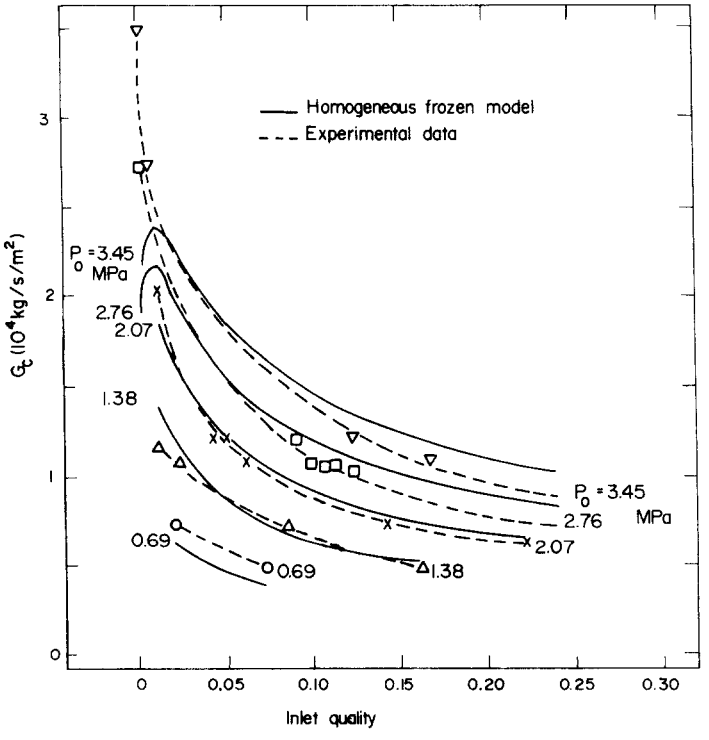

Figure 6(d).

Figure 6. Comparison between several simple models and a single set of data (Hutcherson).

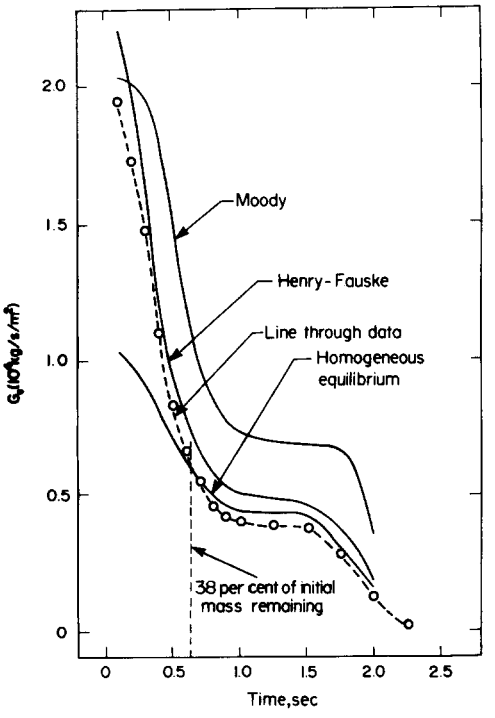

Figure 7(a).

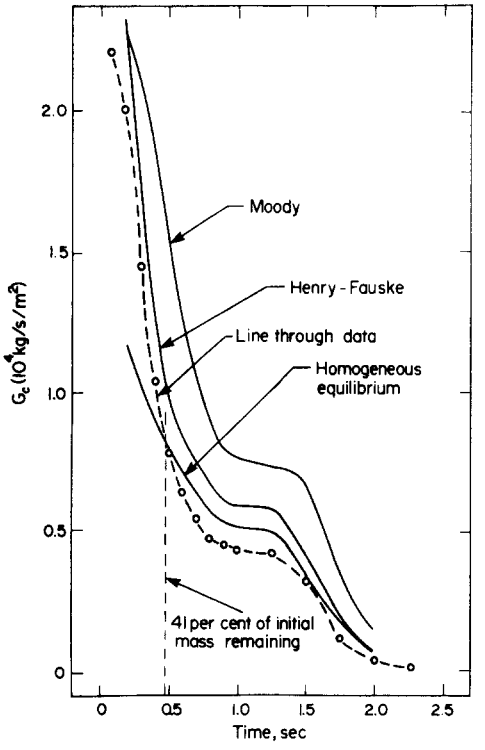

Figure 7(b).

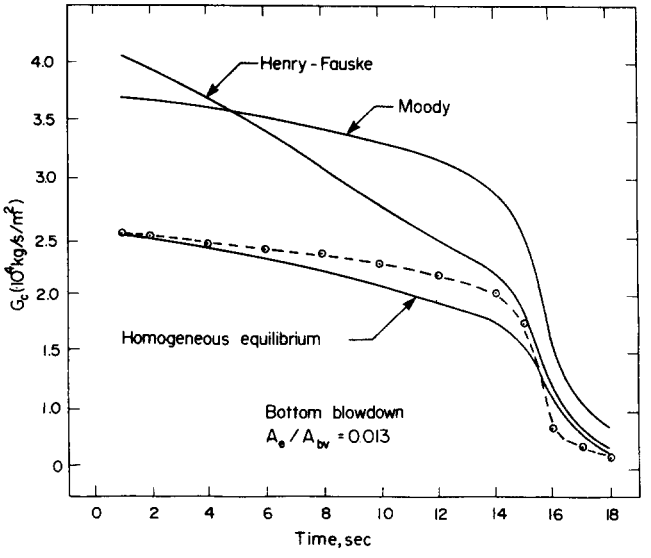

Figure 7(c).

Figure 7. Comparison between predictions and blowdown data showing agreement with the Henry–Fauske model during part of the transient (Hutcherson).

A "relaxation" model for phase change has been proposed by several authors (Kroeger 1976; Bauer *et al.* 1976). The basic idea is that the rate of change of quality is proportional (by means of some empirical coefficient) to the difference between the actual quality and the equilibrium quality; or, alternatively that the vapor generation rate is proportional to some measure of the departure from equilibrium (Stuhmiller 1977). The necessary coefficient may be correlated as a function of steam conditions. This added empirical flexibility allows good representation of data such as Réocreux's (1976, 1977) (e.g. figure 8).

Physically-based models for thermal non-equilibrium

The processes to be represented consists of bubble nucleation and interfacial phase change.

Nucleation. Bubbles can nucleate from walls or from the bulk of the liquid or by entrainment from other parts of the system where they were formed. Changing the nucleation characteristics (e.g. by introducing a grid that causes cavitation) can have considerable influence on the critical flow characteristics (Réocreux 1977).

Although various authors make various assumptions about the source of bubbles (e.g. torn from walls: Andreev *et al.* 1977; growing from nuclei within the liquid by some form of modified kinetic theory: Simpson & Silver 1962; Edwards 1968; Ardron & Ackerman 1978) none is able to escape entirely from empiricism. It is either blatant (e.g. assuming a density of bubble nuclei of a certain initial radius) or hidden somewhere in the algebraic derivations. The numbers are chosen to correlate the resulting critical flow behavior rather than to represent measured nucleation characteristics. Moreover, what evidence there is suggests that actual bubble populations may be orders of magnitude lower than is assumed (Ardron & Ackerman 1978).

Thus, the increase in realism gained by incorporating this process into the analysis is largely offset by uncertainties about the quantitative mechanisms involved. Additional empiricism may be introduced by assuming some delay time before bubbles can start to grow (Edwards 1958).

Vapor generation

Many authors have modeled vapor generation as a bubble growth process in the Plesset & Zwick (1954), Forster & Zuber (1954) tradition. Once the initial inhibiting influences of surface tension are overcome the rate of growth is assumed to be governed by transient conduction in the liquid surrounding each bubble (Simpson & Silver 1962; Edwards 1968; Ardron 1978; Wolfert 1976). Since the effects of neighboring bubbles are ignored and a "thermal boundary layer" approach is often used the method should only be valid at very low qualities.

The rates of bubble growth predicted by the usual transient conduction theories are almost certainly too low. Apart from the effect of decreasing pressure (Zuber & Jones 1976) it can be shown from an order of magnitude analysis that the large pressure gradients and accelerations

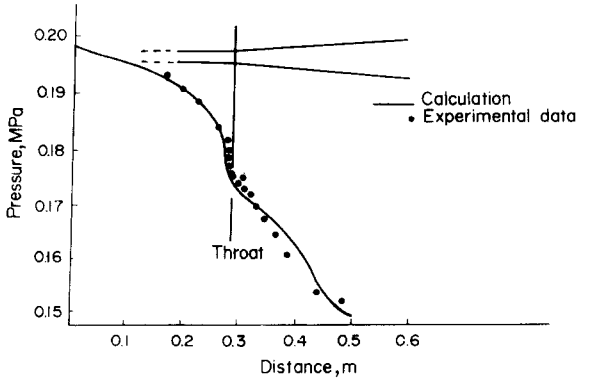

Figure 8. Fitting Réocreux's data with the EDF model using an empirical relaxation representation of phase change.

in rapid expansions produce relative motion of the bubbles in the liquid such that during most of the expansion the convective contribution to heat transfer dominates the transient conduction component (Wolfert 1976; Wallis *et al.* 1976). A similar conclusion is reached in recent work by Ardron (1978).

An earlier analysis of vapor generation by transient conduction on the surface of a liquid jet by Silver (1948) also probably underestimates the vapor generation rate (Chen & Isbin 1966); the predicted initial flow rates are perhaps reasonable because of the concurrent assumption of equal phase velocities, inconsistent with the assumed separated flow model.

My overall impression of the various theories for vapor generation is that so far they represent no improvement over the grosser empirical methods (such as the Henry and Fauske model). There are so many uncertainties about the physics that one merely ends up trying to fit critical flow data by varying adjustable coefficients that are made respectable by being given names implying that they describe physical phenomena.

If any improvement to this situation is to be achieved we need a concerted effort to measure independently the various components of the model (such as bubble size, interphase area and heat transfer, number of nuclei of a given size) and evolve a theory that is compatible with this detailed evidence. This may be possible in a very well controlled laboratory environment. There still remains doubt in my mind as to whether in a practical application we will ever be in a position to know the upstream conditions well enough to justify this level of detail in the analysis.

Two-fluid (separated flow) models

A mathematical framework that appears in principle capable of representing most of the recognised non-equilibrium phenomena is the "two-fluid" or "separated flow" model (Ishii 1975; Bouré 1977; Wendroff 1979). Separate conservation equations are written for each phase (or for one phase and the combined phases separately) and interaction terms are included to represent the interphase heat, mass and momentum transfer. Upstream conditions, including a description of the nucleation centers, are supplied as one boundary condition and the solution is developed numerically in the downstream direction. Critical flow occurs when the determinant of the coefficients of the solution variables is zero at a location, such as the exit plane, where the numerator that is to be divided by this determinant can also be arranged to be zero. A very thorough description of the general mathematics of this solution technique is given in several papers by Bouré *et al.* (1975, 1977).

Though critical flow is achieved as a local mathematical singularity, it is approached by solving the entire history of the flow and cannot be predicted without performing this chore. Of course, when this is done there are several ways besides recording the critical flow rate by which the success of the analysis can be checked (e.g. pressure and void fraction measurements along the duct).

The problem with this approach is knowing what coefficients and functions to put into the basic equations to represent the interactions between the phases. To describe the interphase drag and apparent mass (virtual inertia, inertia coupling) effects, for example, we need to know at least the flow pattern (e.g. bubble flow) and entity size (e.g. bubble volume). We need to assume something about the applicability of steady state drag correlations in an accelerating flow, and about the effects of interface (e.g. bubble) size and shape changes on those forces. New terms that depend on the rate of change of flow properties or spatial gradients in parameters (such as void fraction) may be needed. Up to now we have very little information about what needs to be put into these equations to make them usable. Moreover, there is no inherent assurance that the set of correlations allowing all of these various terms to be computed may not turn out to be more cumbersome, complicated and unreliable than merely correlating critical flow data by adding a couple more empirical coefficients to an existing, reasonably good computational tool such as the Henry and Fauske model.

An argument in favor of developing this technique for representing critical flow for the particular application of accident analysis for light water nuclear reactors is that there is a trend towards the use of two-fluid models for the entire circuit. If this attempt succeeds, critical flow may turn out to be a special case that is handled routinely by the universal calculation procedure.

Before giving an indication of how well this approach works it is worth mentioning attempts to reduce the full two-fluid model to a smaller number of equations. One approach has been the use of the "drift flux" to characterise the relative motion (e.g. Kroeger 1976). The hope is that the drift flux can be determined from some independent correlation and not through the solution of the equations. This particular approach is ill-advised. The relative motion in a rapidly accelerating flow with changing void fraction (and flow pattern?) is determined by a quite different set of terms in the momentum balance than would obtain under the conditions from which the correlation was derived. One really needs another mechanistically-based equation to determine the drift flux; as a result nothing has been gained unless the mathematical solution is in some way simplified. I believe Bouré is wise to recommend caution in using "incomplete models" of this type (Bouré 1977).

An example of the use of this approach is the recent work of Richter (1979). He uses six conservation equations. For a given flow regime (e.g. bubble flow) he uses correlations for interphase and wall friction and heat transfer that were derived in much simpler circumstances. He suggests an empirical function of void fraction to modify the single bubble drag correlation and chooses a particular representation of apparent mass effects. He calculates the rate of vapor formation from the energy equations. In order to "fit" experimental data he needs to adjust the "nucleation site density", N_i, and the size of these nuclei, d_i, that determine the superheat necessary for them to start growing. He can fit many results in the literature (e.g. figures 9 and 10) by assuming an initial bubble diameter of 25 μ and a density of 10^{11} nuclei/m^3. For some systems other assumptions are better (e.g. figure 11).

Is this method better than simpler approaches that can fit similar data by using two adjustable factors? I don't think we are yet in a position to say. The only way we will know is when there is a sufficient base of detailed measurements from which the components of the model (nucleation characteristics, interphase transfer laws) can be checked *independently*. This will require the sort of thorough tenacious study conducted by Réocreux (1977) with the addition of instrumentation to measure quantities such as interphase area, individual phase temperatures, local phase velocities or other parameters that can be used to give a fair

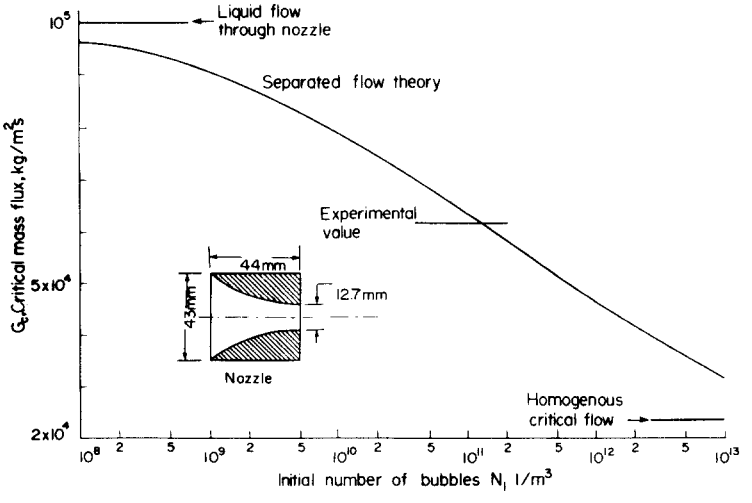

Figure 9. Effect of nucleation site density on prediction of short nozzle data of Sozzi–Sutherland.

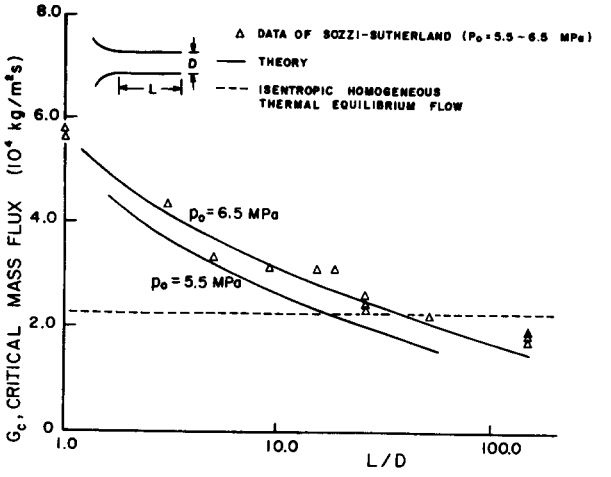

Figure 10. Effect of nozzle length on critical mass flux predicted by the separated flow model ($N_i = 10^{11}$ m^{-3}, $d_i = 25\ \mu$).

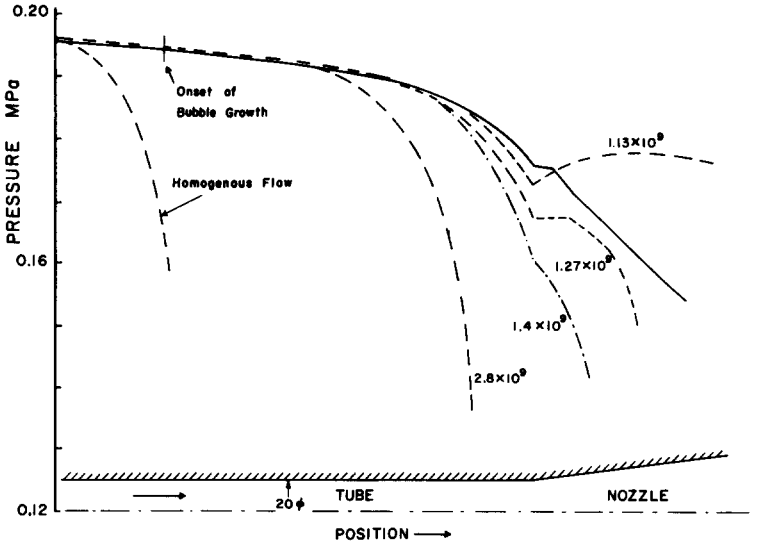

Figure 11. Attempts to match the pressure profile in one of Réocreux's experiments by varying the nucleation site density (N_i). (P_{sat} = 0.21 MPa, d_i = 18 μ) ——— experimental, — — — theory.

comparison between theory and experiment. Perhaps this sort of information can only be gathered under very carefully controlled laboratory conditions.

This is the point where I perceive a need to achieve a sense of proportion. I believe it is a good rule that the sophistication of a theoretical analysis should match the degree to which the physical phenomena can be specified. I am not sure that the nucleation characteristics and upstream flow conditions (such as phase temperatures, quality and flow pattern) in a practical system are usually sufficiently well known to allow very precise definition of the problem. In the case of a loss-of-coolant accident the location and shape of the "break" are infinitely variable parameters. Yet small changes (e.g. 0.1 per cent in quality in figure 2) can have effects on flow rate that are comparable with the degree of improvement in accuracy that might be gained by using the most complex theories rather than the simplest ones.

CONCLUSIONS

Some simple models for critical flow exist. They can be modified empirically to fit data.

More elaborate models, including analysis of non-equilibrium phenomena, require empirical correlation of phase interactions or the making of assumptions that are so far untested.

For practical purposes some balance has to be struck between these levels of complexity. Good judgement suggests that the appropriate analytical approach should be consistent with our understanding of the physical phenomena and the precision with which they can be defined.

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